RSA Digital Signature Standards

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Outline

I. Background

II. Forgery and provable security

III. Contemporary signature schemes

IV. Standards strategy

Part I: Background

General Model

- A signature scheme consists of three (or more) related operations
- *Key pair generation* produces a public/private key pair
- *Signature operation* produces a signature for a message with a private key
- *Verification operation* checks a signature with a public key

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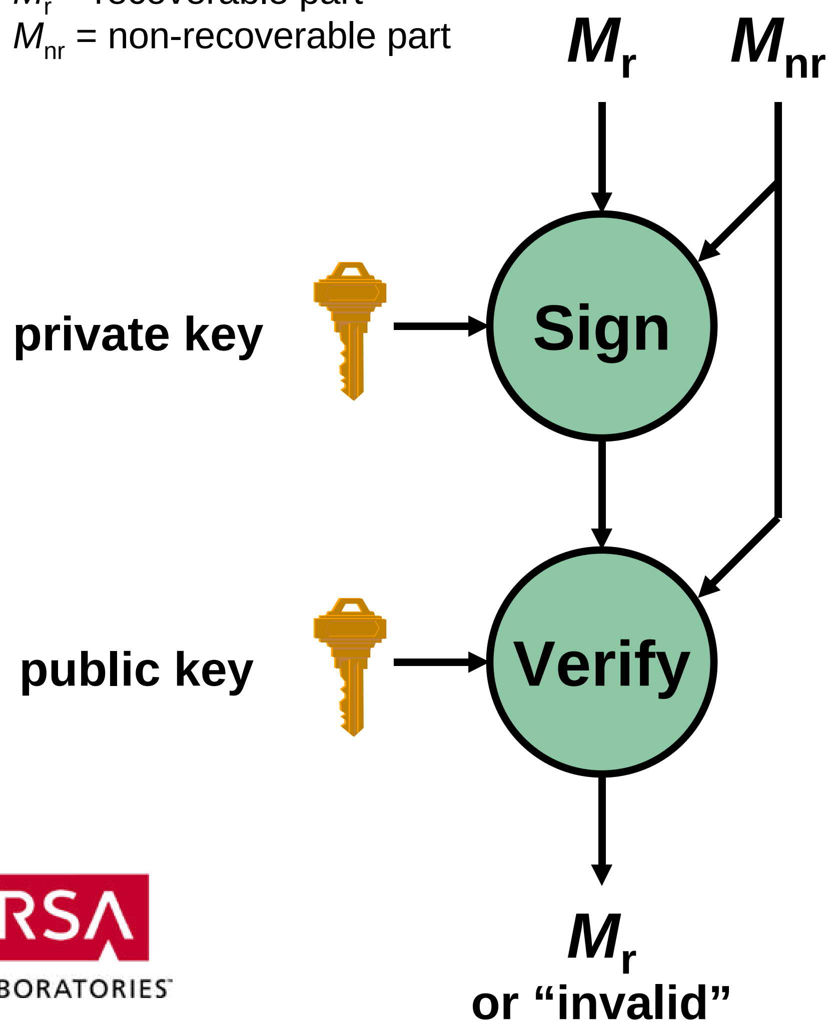
Types of Signature Scheme

M_r = recoverable part
M_{nr} = non-recoverable part



- ***Appendix:*** **message transmitted with signature**
- ***Total message recovery:*** **message recoverable from signature**
- ***Partial message recovery:*** **part of message recoverable from signature, part transmitted**

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Trapdoor One-Way Functions

- A *one-way function* $f(x)$ is easy to compute but hard to invert:
  - easy: $x \rightarrow f(x)$
  - hard: $f(x) \rightarrow x$
- A *trapdoor* one-way function has trapdoor information f^{-1} that makes it easy to invert:
  - easy: $f(x), f^{-1} \rightarrow x = f^{-1}(f(x))$
- Many but not all signature schemes are based on trapdoor OWFs

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RSA Trapdoor OWF

- The RSA function is

$$f(x) = x^e \bmod n$$

where $n = pq$, p and q are large random primes, and e is relatively prime to p-1 and q-1

- This function is conjectured to be a trapdoor OWF
- Trapdoor is

$$f^{-1}(x) = x^d \bmod n$$

where $d = e^{-1} \bmod \mathrm{lcm}(p-1, q-1)$

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Embedding Operations

- An embedding operation $\mu(M)$ maps from message strings to "message representatives," which can be input to f^{-1}
  - e.g., a hash function with padding
  - may be randomized
- Inverse operation checks whether a message representative is correct
  - in scheme with message recovery, also recovers message part
- Current RSA signature schemes differ primarily in terms of the embedding operation

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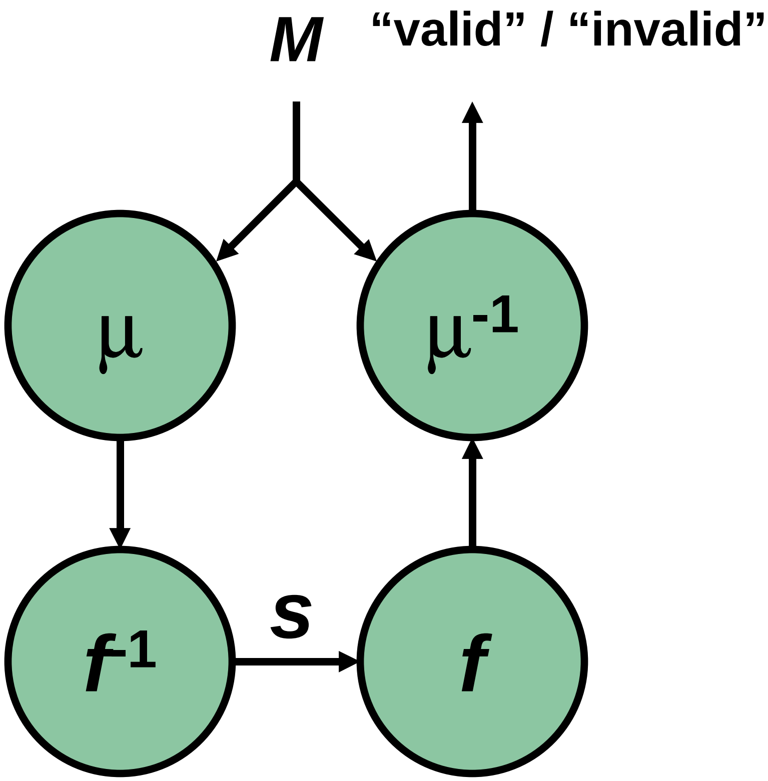
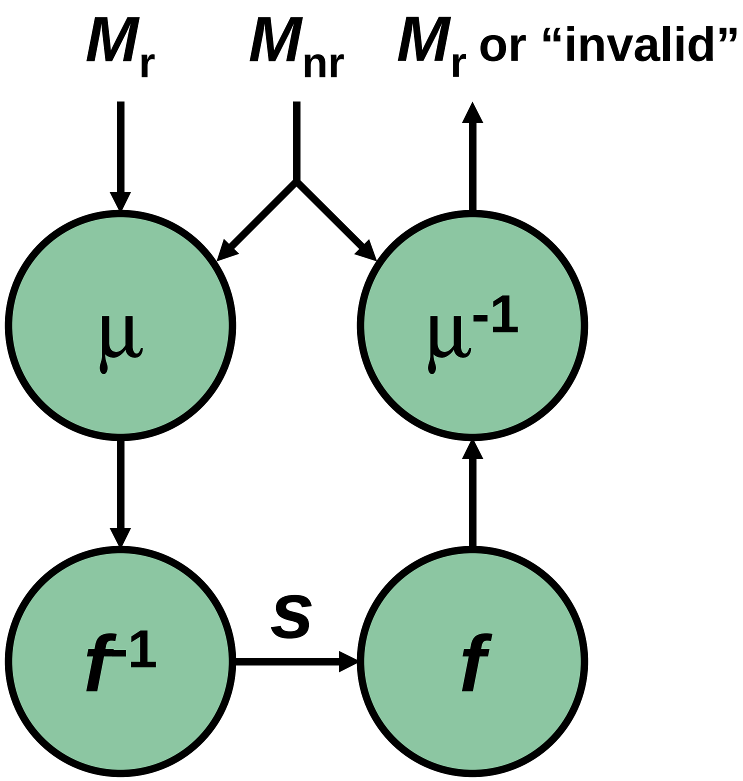
Scheme with Appendix



- Signature generation embeds message, applies trapdoor:
  - $s = f^{-1}(\mu(M))$
- Signature verification applies OWF, checks against message:
  - $\mu^{-1}(f(s), M)$ valid?

Scheme with Message Recovery



- Signature generation embeds message, applies trapdoor:
  - $s = f^{-1}(\mu(M_r, M_{nr}))$
- Signature verification applies OWF, checks against M_{nr}, recovers M_r:
  - $M_r = \mu^{-1}(f(s), M_{nr})$

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Embedding Properties

- **Embedding operation should have similar properties to a hash function:**
  - one-way: for random x, hard to find M s.t. $\mu(M) = x$
  - collision-resistant: hard to find M_1, M_2 s.t. $\mu(M_1) = \mu(M_2)$
- **May also identify underlying algorithms**
  - but if so, must be done with care
- **Should also interact well with trapdoor function**
  - ideally, mapping should appear "random"

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Multiplicative Properties of RSA

- **RSA function is a *multiplicative homomorphism:* for all x, y,**

$$f\,(xy \bmod n) = f(x)\,f(y) \bmod n$$

$$f^{-1}(xy \bmod n) = f^{-1}(x)\,f^{-1}(y) \bmod n$$

- **More generally:**

$$f^{-1}\left(\prod x_i \bmod n\right) = \prod \left(f^{-1}(x_i)\right) \bmod n$$

- **Property is exploited in most forgery attacks on RSA signatures, but also enhances recent security proofs**

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Part II: Forgery and Provable Security

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Signature Forgery

- **A *forgery* is a signature computed without the signer's private key**
- **Forgery attacks may involve interaction with the signer: a *chosen-message* attack**
- **Forgery may produce a signature for a specified message, or the message may be output with its signature (*existential forgery*)**

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Multiplicative Forgery

- **Based on the multiplicative properties of the RSA function, if**

$$\mu(M) = \prod \mu(M_i)^\wedge\alpha_i \bmod n$$

then

$$\sigma(M) = \prod \sigma(M_i)^\wedge\alpha_i \bmod n$$

- **Signature for *M* can thus be forged given the signatures for $M_1, \ldots, M_l$ under a chosen-message attack**

Small Primes Method

- Suppose $\mu(M)$ and $\mu(M_1)$, …, $\mu(M_l)$ can be factored into small primes
  - Desmedt-Odlyzko (1986); Rivest (1991 in PKCS #1)
- Then the exponents α_i can be determined by relationships among the prime factorizations
- Requires many messages if μ maps to large integers, but effective if μ maps to small integers
- Limited applicability to current schemes

Recent Generalization

- **Consider $\mu(M)$, $\mu(M_1)$, …, $\mu(M_l)$ mod n, and also allow a fixed factor**
  - Coron-Naccache-Stern (1999)
- **Effective if μ maps to small integers mod n times a fixed factor**
- **Broader applicability to current schemes:**
  - ISO 9796-2 [CNS99]
  - ISO 9796-1 [Coppersmith-Halevi-Jutla (1999)]
  - recovery of private key for Rabin-Williams variants [Joye-Quisquater (1999)]

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Integer Relations Method

- **What if the equation**

$$\mu(M) = f(t) \prod \mu(M_i)\text{^}\alpha_i$$

  could be solved without factoring?

- **Effective for *weak* μ**
- **ISO 9796-1 broken with *three* chosen messages [Grieu (1999)]**

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Reduction Proofs

- A *reduction proof* shows that inverting the function *f* "reduces" to signature forgery: given a forgery algorithm *F*, one can construct an inversion algorithm *I*
- *Provable security*: inversion hard $\rightarrow$ forgery hard
- "Tight" proof closely relates hardness of problems

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Random Oracle Model

- **In the *random oracle* model, certain functions are considered “black boxes”: forgery algorithm cannot look inside**
  - **e.g., hash functions**
- **Model enables reduction proofs for generic forgery algorithms — inversion algorithm hides value to be inverted in oracle outputs**
- **Multiplicative properties of RSA can enhance the proof**

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Part III: Contemporary Signature Schemes

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Overview

- Several popular approaches to RSA signatures
- Approaches differ primarily in the mapping μ
- Some differences also in key generation
- Some also support Rabin-Williams (even exponent) signatures

- There are many other signature schemes based on factoring (e.g., Fiat-Shamir, GQ, Micali, GQ2); focus here is on those involving the RSA function

Schemes with Appendix

- Basic scheme
- ANSI X9.31
- PKCS #1 v1.5
- Bellare-Rogaway FDH
- Bellare-Rogaway PSS
- IEEE P1363a version of PSS

Basic Scheme

- $\mu(M) = \text{Hash}(M)$
- Pedagogical design
- Insecure against multiplicative forgery for typical hash sizes
- (Hopefully) not widely deployed

ANSI X9.31

(Digital Signatures Using Reversible Public-Key Cryptography for the Financial Services Industry, 1998)

- $\mu(M)$ = 6b bb … bb ba || Hash(M) || 3x cc

  where x = 3 for SHA-1, 1 for RIPEMD-160
- Ad hoc design
  - cc octet for RW support
- Resistant to multiplicative forgery
  - some moduli are more at risk, but still out of range
- Widely standardized
  - IEEE 1363, ISO/IEC 14888-3
  - US NIST FIPS 186-1
- ANSI X9.31 requires “strong primes”

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PKCS #1 v1.5

(RSA Encryption Standard, 1991)

- $\mu(M)$ = 00 01 ff … ff 00 || HashAlgID || Hash(M)
- Ad hoc design
- Resistant to multiplicative forgery
  - moduli near 2^k are more at risk, but still out of range
- Widely deployed
  - SSL certificates
  - S/MIME
- Included in IEEE P1363a; PKCS #1 v2.0 continues to support it

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ANSI X9.31 vs. PKCS #1 v1.5

- Both are deterministic
- Both include a hash function identifier
- Both are ad hoc designs
  - both resist [CNS99]/[CHJ99] attacks
- Both support RSA and RW primitives
  - see IEEE P1363a contribution on PKCS #1 signatures for discussion
- No patents have been reported to IEEE P1363 or ANSI X9.31 for these mappings

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Bellare-Rogaway FDH

(Full Domain Hashing, ACM CCCS ’93)

- $\mu(M)$ = Full-Length-Hash(m)
- Provably secure design
  - resists any attack where hash function is considered a black box, provided that RSA is hard to invert
- Variant included in IEEE P1363a, PKCS #1 v2.1 draft

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Bellare-Rogaway PSS

(Probabilistic Signature Scheme, Eurocrypt '96)

- $\mu(M) \approx H \,||\, G(H) \oplus salt$

  where H = Hash($salt$, M), $salt$ is random, and G is a mask generation function

- **Provably secure design**
- **Variant included in IEEE P1363a, PKCS #1 v2.1 draft**

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FDH vs. PSS

- FDH is deterministic, PSS is probabilistic
- Both are provably secure designs
  - same paradigm as Optimal Asymmetric Encryption Padding (OAEP)
- PSS has tighter security proof, is less dependent on security of hash function
- PSS-R variant supports message recovery, partial message recovery
- PSS is patent pending (but generously licensed)

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IEEE P1363a Version of PSS

- $\mu(M) = G(H) \oplus [00 \ldots 01 \,||\, salt] \,||\, H \,||\, \mathrm{bc}$

  where $H \approx \mathrm{Hash}(salt, \mathrm{Hash}(M))$, *salt* is random, and G is a mask generation function

- Salt combined with Hash(*M*) rather than *M* for practical and security reasons:
  - "single-pass" processing
  - provable security if Hash(*M*) outside crypto module
  - protection against fault-analysis attacks

- Salt can be omitted for FDH-like scheme

Schemes with Message Recovery

- Basic scheme
- ISO/IEC 9796-1
- ISO/IEC 9796-2
- Bellare-Rogaway PSS-R
- IEEE P1363a version of PSS-R

Basic Scheme

- $\mu(M_r) = M_r$
- Another pedagogical design (“textbook RSA”)
- Insecure against various forgeries, including existential forgery
  - attacker can select signature s then “recover” $M_r = f(s)$
- Again, hopefully not widely deployed

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ISO/IEC 9796-1

(Digital Signature Scheme Giving Message Recovery, 1991)

- $$\mu(M_r) = \pi^*(m_{l-1})\ \pi'(m_{l-2})\ m_{l-1}\ m_{l-2}\ \pi(m_{l-3})\ \pi(m_{l-4})\ m_{l-3}\ m_{l-4} \ldots \pi(m_3)\ \pi(m_2)\ m_3\ m_2\ \pi(m_1)\ \pi(m_0)\ m_0\ 6$$

  where m_i is the ith nibble of M_r and π^*, π' and π are permutations

- Ad hoc design with significant rationale
- *Not* resistant to multiplicative forgery [CHJ99] [Grieu 1999]
  - may still be appropriate if applied to a hash value
- Moderately standardized

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ISO/IEC 9796-2

(Digital Signature Scheme Giving Message Recovery — Mechanisms Using a Hash Function, 1997)

- $\mu(M_r, M_{nr}) \approx$ 6a || M_r || H || bc
- $\mu(M_r)$ = 4b bb … bb ba || M_r || H || bc

  where H = Hash (M_r, M_{nr}) or Hash (M_r)
  - (assumes modulus length is multiple of 8)
  - general format allows hash algorithm ID
- Ad hoc design
- *Not* resistant to multiplicative forgery if hash value is 64 bits or less [CNS99]
  - may still be appropriate for larger hash values
- Newly standardized

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Bellare-Rogaway PSS-R

(Probabilistic Signature Scheme with Recovery, 1996)

- $\mu(M_r, M_{nr}) \approx H \ || \ G(H) \oplus [salt \ || \ M_r]$

  where H = Hash($salt$, M_r, M_{nr}), $salt$ is random, and G is a mask generation function

- Provably secure design
- Variant included in IEEE P1363a, draft revision of ISO/IEC 9796-2

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IEEE P1363a Version of PSS-R

- $\mu(M_r, M_{nr}) = G(H) \oplus [00 \ldots 01 \,||\, M_r \,||\, salt] \,||\, H \,||\, \mathrm{bc}$

  where $H \approx \mathrm{Hash}(salt, M_r, \mathrm{Hash}(M_{nr}))$, *salt* is random, and G is a mask generation function
- Extension of PSS variant
  - PSS variant is special case where M_r is null

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Part IV: Standards Strategy

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Standards vs. Theory vs. Practice

- **ANSI X9.31 is widely standardized**
- **PSS is widely considered secure**
- **PKCS #1 v1.5 is widely deployed**

- **How to harmonize signature schemes?**
  - **(primary question for signature schemes with appendix; related question for message recovery)**

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Challenges

- **Infrastructure changes take time**
  - **particularly on the user side**
- **ANSI X9.31 is more than just another encoding method, also specifies “strong primes”**
  - **a controversial topic**
- **Many communities involved**
  - **formal standards bodies, IETF, browser vendors, certificate authorities**

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Prudent Security

- What if a weakness were found in ANSI X9.31 or PKCS #1 v1.5 signatures?
  - no proof of security, though designs are well motivated, supported by analysis
  - would be surprising — but so were vulnerabilities in ISO/IEC 9796-1,-2
- PSS embodies “best practices,” prudent to improve over time

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Proposed Strategy

- **Short term (1-2 years): Support both PKCS #1 v1.5 and ANSI X9.31 signatures for interoperability**
  - e.g., in IETF profiles, FIPS validation
  - FIPS 186-2 schedule allows PKCS #1 v1.5 for an 18-month transition period, FPKI TWG is requesting a further extension
- **Long term (2-5 years): Move toward PSS**
  - upgrade in due course — e.g., with AES algorithm, new hash functions
  - separate *assurance* requirements from interoperability
    - e.g., key sizes, key protection, “strong primes”

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Standards Work

- PSS, PSS-R standardization work in progress in various forums:
  - IEEE P1363a
  - PKCS #1 v2.1
  - ISO/IEC 9796-2 revision
- Coordination ongoing, ballot target Spring 2001
- Promotion in other forums planned
  - ANSI X9.31
  - FIPS
  - IETF

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Conclusions

- Several signature schemes based on RSA algorithm
  - varying attributes: standards, theory, practice
- Recent forgery results on certain schemes, security proofs on others
- PSS a prudent choice for long-term security, harmonization of standards